Dynamic Effects of Trade Policy and Capital Controls: A Terms-of-Trade Manipulation Perspective

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Abstract

This paper studies the interaction between trade policy and capital control policy from a terms-of-trade manipulation perspective. I extend the dynamic two-country multi-good endowment economy in Costinot, Lorenzoni and Werning (2014) with trade taxes/subsidies. Home country chooses optimal taxes on all tradable goods and international capital flows in order to maximize domestic welfare, while the foreign country is passive. When only good-specific trade taxes/subsidies are available, Home has incentive to manipulate tariffs to depreciate its real exchange rate, if it has faster growing endowment than the Foreign. Moreover, I find that taxing capital inflow/subsidizing capital outflow is equivalent to a uniform reduction in gross trade tariffs on all goods. If the government can get access to good-specific tariffs/subsidies, there is no need for capital controls.

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1 Introduction

Based on a motive of manipulating terms-of-trade, this paper presents a dynamic analysis of two distinct policy instruments: trade taxes/subsidies and capital flow taxes. For trade protection or some political reasons, trade taxes/subsidies are natural instruments to manipulate term-of-trade. The analysis on optimal trade policy can date back to the nineteenth century, and recently received a lot of attention due to the ongoing U.S.-China trade war. However, since existing literature mainly focus on static models and rely on a strong balanced trade assumption, researchers usually neglect tariffs' intertemporal effects on terms-of-trade. Then, evaluating tariffs' effects under a dynamic macro environment comes to mind. The second policy instrument, capital controls, which is a capital account policy often used to manage exchange rate or prevent a FX crisis, could also distort equilibrium prices, similar as an indirect trade policy tool that would affect on all sectors. Recently, some literature reveal that capital control policy can be used to manipulate the intra or intertemporal terms of trade, conceptually analogous to the traditional instrument – trade tariffs to manipulate the goods' terms-of-trade (for example, Costinot, Lorenzoni and Werning 2014, henceforth, **CLW**, Heathcote and Perri 2016, and Brunnermeier and Sannikov 2015). The questions then arise: when both policy instruments were available, what would the optimal policy look like? Furthermore, how would the trade policy compare with capital controls?

My starting point is the two-country endowment economy in CLW (2014), in which they show that a country might use capital controls as a form of intertemporal trade policy tools to manipulate interest rate in its favor. A country with faster growing endowment would like to tax capital inflow or subsidize capital outflow to encourage domestic saving. In my paper, under the same environment with multiple goods, I first characterize unilaterally optimal tariffs, and then compare the impact of trade policy with capital controls. I focus on this model because it provides a fundamental theory of capital controls for the same purpose as tariffs – terms-of-trade manipulation. Such dynamic settings enable us to analyze the interaction between these two policies at intra- and inter-temporal levels.

The main results are stated as follows. First, I find that the unilaterally optimal trade taxes would support its real exchange rate in its own favor. For example, if Home has a faster growth rate than the foreign country, it has incentive to use trade taxes to depreciate its real exchange rate. Second, by focusing a simple two-good example with constant relative risk aversion (CRRA) utility and constant elasticity of substitution (CES) demand, I find that the optimal tariffs on both sectors are identical. Third, when both of tariffs and capital flow taxes can be imposed unilaterally, capital controls are partial substitute of trade tariffs. The CLW's optimal capital control policy, taxing capital inflow/subsidizing capital outflow when there is an endowment growing, is equivalent to the trade policy of reducing the gross tariffs/increasing the gross subsidies on all sectors uniformly. It means the impact of trade taxes/subsidies can offset capital controls if they affect the real exchange rate in the opposite way. Last but not the least, when optimal good-specific trade taxes are available, there is no need to tax on international capital flows.

The intuition of the dynamic effects of trade taxes/subsidies and capital controls is as follows. When Home has a larger trade deficits, as a net buyer, it has a stronger incentive to lower the price by decreasing its domestic consumption. Similarly, larger trade surpluses would give Home more incentive to raise world price by stimulating domestic consumption. Since faster growing at home means either less trade deficits or larger trade surplus tomorrow, Home thus has stronger incentives to stimulate future consumption relative to current consumption. To subsidize future domestic consumption, Home can either encourage current domestic saving by imposing a tax on capital inflow or subsidizing capital outflow, or by depreciating future real exchange rate by adjusting tariffs rate.

In terms of methodology, I follow closely CLW (2014), who use the primal approach to first characterize first optimal wedges, and then implement the optimal allocations with capital flow taxes. Since typically there are many ways to achieve the optimal implementation, primal approach facilitates the analysis of equivalence of different policy instruments. This paper considers the unilaterally optimal policy in two forms: trade taxes/subsidies which are imposed on the excess demand/supply of all tradable goods, and capital control policy in a form of tax on the international capital flows. The result later shows that both policy instruments can be used as alternative policy tools to manipulate terms-of-trade, interest rate, or indirectly real exchange rate in its favor.

Related Literature

This paper relates to two different strands of the literature: optimal trade policy, and capital controls. There are large and varied literature on optimal tariffs in the international trade. However, most of trade policy papers assume that trade is balanced under a static model. This strong assumption is relaxed automatically when we consider the trade taxes in a dynamic setting. A country can run an unbalanced current account today through lending or borrowing in the international financial market. In terms of the scope of impact on terms-of-trade, capital controls is a blunter policy instrument that would affect all sectors, while tariffs can be tailored to specific sectors.

The Literature on Optimal Trade Policy Based on the idea of term-of-trade manipulation, many theoretical and empirical analysis of optimal tariffs have been done to reveal the impact and implications of trade agreement and WTO (see Bagwell and Staiger 1999, and Broda, Limao and Weinstein 2008). Various of studies of optimal trade policy demonstrate the importance of terms-of-trade manipulation from different angles. Costinot, Donaldson, Vogel and Werning (2015) exploring the optimal trade policy from the traditional theory of comparative advantage. Costinot, Rodriguez-Clare and Werning (2016) consider the optimal trade policy implication from firm heterogeneity. Beshkar and Lashkaripour (2019) studies optimal trade policy structure in the presence of various cross-industry linkages, and interdependencies among various trade policy instruments.

Contrary to the conventional view, Lake and Linask (2016) find the tariffs in fact are pro-cyclical due to the pro-cyclical market power in developing countries. However, my result shows that the optimal capital controls in CLW resemble counter-cyclical tariffs.

As far as I know, no previous studies examined the optimal trade policy in a dynamic setting, although the studies on international trade policy has a very long history. The goal of this paper is to take a first stab at exploring the optimal tariffs behavior in the dynamic setting and making a comparison of tariffs and capital controls in terms of manipulating terms-of-trade.

The Literature on Capital Controls This paper is also related to a recent and growing literature about the use of capital controls. Unlike tariffs came under widespread criticism, the landscape of the capital control debate has changed in the past decade.[1] After the global financial crisis, the International Monetary Fund changed its orthodox views on capital controls, proposing a appropriate use of capital controls to prevent financial crisis.[2]

Capital control policy can induce favorable changes in international interest rates, terms-of-trade, or both that are beneficial for the country that puts the controls in place. Heathcote and Perri (2016) shows that capital control policy is desirable in a standard two-country international business cycle model, since it can generate favorable changes in equilibrium interest rate and terms-of-trade, while at the expense of foreign country.

The mainstream thoughts on desirability of capital controls focus on financial stability. A pervasive pecuniary externalities view takes capital controls as a Pigouvian tax that internalizes the external effects of households' over-borrowing behavior. For instance, Benigno, Chen, Otrok, Rebucci and Young (2016) studies the use of capital controls and real exchange rate policy to prevent financial crisis. They show when real exchange policy (explicitly shown as consumption taxes on tradable and non-tradable goods) is costless, there is no need for restricting international capital flows. Moreover, if exchange rate policy is costly, optimal mix combines prudential capital controls. My result resembles theirs. However, in their paper, the only rationale for capital controls is financial stability, instead of manipulating the terms-of-trade or other relative prices. Brunnermeier and Sannikov (2015) shows that capital controls as a macro-prudential tool, can improve the welfare in an economy with multi-good, incomplete financial markets, and inefficient production,

[1] Ghosh and Qureshi (2016) investigates why capital controls on inflows have a "bad" name by delving into historical record dating back to the gold standard era.

[2] See Ostry et al. (2010) and IMF (2012).

but not trade policy of taxes/subsidies.

Few literature made an attempt to link capital account policy, trade tariffs and real exchange rate. Jones (1967) sets forth a static two-country two final goods model to analyze the optimal tariff policy with mobile capital. However, he treats international capital flows similar as a third intermediate good. Therefore, there is no intertemporal international capital flow. Edwards (1989) analyzes how capital controls affect the terms-of-trade and the real exchange rate based on a stylized model. Jeanne (2012) shows a equivalence of capital controls and standard trade protectionist measures, such as a import tariff and a export subsidy. In his paper, however, there is only one tradable good, so that intra-temporal effects from tariffs are neglected.

The rest of the paper is structured as follows. Section 2 represents the baseline CLW model in the presence of trade taxes/subsidizes. In the first subsection, following CLW with primal approach and largrangian method, I show the pro-cyclical behavior of domestic aggregate consumption when there is no price distortion. I then characterize the unilaterally optimal tariff policy. For illustration, I further provide an example with CRRA and asymmetric CES utility, which results in a uniform optimal tariff on both sectors. Section 3 presents the interaction between trade policy and capital controls by implementing the optimal allocations with both policy instruments. Section 4 concludes. Proofs of Propositions are collected in the Appendix.

2 Model

2.1 Basic Environment: Multi-good Economy in CLW (2014)

It is helpful to start with the setup of the multiple-good economy in CLW (2014). With primal approach, I assume that Home government can choose the equilibrium allocations directly to maximize Home's consumer's welfare. Along the optimal path, it determines how the domestic consumption vary with home endowment change, and further implies the change of first optimal wedge. The implementation of optimal allocations with explicit policy instruments will be addressed in the later subsection.

Assume that there are two countries, Home and Foreign in the multi-good economy. All $n > 1$ goods are tradable. The representative consumer at Home faces a maximization problem.

$$\max_{\{c_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t U(C_t)$$

where $\beta \in (0,1)$ is the discount factor, utility function U is twice continuously differentiable, strictly increasing, strictly concave and satisfies Inada conditions, $C_t \equiv f(c_t)$ is the aggregate domestic consumption at date t, $c_t \in \mathbb{R}^n_+$ is the domestic consumption vector, and f is increasing, concave, and homogeneous degree of one. The utility functions of representative consumer at foreign country has a similar form. Analogous definitions apply to foreign country's variables, U^* and $C_t^* \equiv f^*(c_t^*)$.

To simplify the model, I make additional following assumptions on endowment. Home receives an endowment sequence, $\{y_t\}$, where $y \in \mathbb{R}^n_+$, and Foreign receives $\{y_t^*\}$, where $y^* \in \mathbb{R}^n_+$. Both endowment, $\{y_t\}$ and $\{y_t^*\}$, $\forall t$, are bounded away from zero and total world endowment is fixed.

I first derive the implementability constraint for Home country from foreign consumer's problem. The foreign consumer solves

$$\begin{aligned} \max_{\{c_t^*\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t U^*(C_t^*) \\ s.to. \quad & \sum_{t=0}^{\infty} p_t^* \cdot (c_t^* - y_t^*) \leq 0 \end{aligned}$$

where $p_t^* \in \mathbb{R}_+^n$ are world price vectors, and $\cdot$ is inner product. The corresponding first order condition is

$$\beta^t U^{*'}(C_t^*) \nabla f_c^*(c^*) = \lambda^* p_t^* \tag{1}$$

where λ^* is the Lagrangian multiplier of the foreign consumer's budget constraint.

Note that without good-specific taxes or subsidies, $\frac{p_{it}^*}{p_{jt}^*} = \frac{f_i(c_t)}{f_j(c_t)} = \frac{f_i^*(c_t)}{f_j^*(c_t)}$, and accordingly, the consumption allocation (c_t, c_t^*) in any period t is Pareto efficient and solves

$$C^*(C_t) = \max_{c,c^*} \{f^*(c^*) \text{ subject to } c + c^* = Y \text{ and } f(c) \geq C_t\} \tag{2}$$

In this way, foreign aggregate consumption C^* and consumption vectors (c, c^*) can be represented by C_t. Therefore, the Home government can choose an aggregate consumption level C_t, as a choice variable, instead of consumption vector c_t.

With the price vector derived from the first order condition (1), I then state Home's planning problem as

$$\max_{\{C_t\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{3}$$

$$s.to. \quad \sum_{t=0}^{\infty} \beta^t \phi(C_t) \cdot (c_t(C_t) - y_t) = 0 \tag{4}$$

where $\phi(C_t) \equiv U^{*'}(C^*(\phi(C_t)))\nabla f^*(c^*(C_t))$. Equation (4) is an implementability constraint. World price is substituted with equation (1). Then, the first order condition associated with Home's planning problem is

$$U'(C_t) = \mu \left\{ \frac{\partial \phi(C_t)}{\partial C_t} \cdot [c(C_t) - y_t] + \phi(C_t) \cdot \frac{\partial c(C_t)}{\partial C_t} \right\} \tag{5}$$

where μ is the Lagrangian multiplier on the implementability constraint. Condition (5) implies a pro-cyclical aggregate consumption along the optimal path. The following proposition describes how the aggregate consumption changes, when the endowment grows and there are multiple goods.

Proposition 1 (Pro-cyclical aggregate consumption)**.** *The domestic aggregate consumption is higher from period t to period $t+1$, $C_{t+1} > C_t$, if and only if $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} > 0$.*

Proposition 1 indicates that whether the domestic aggregate consumption grows or not depends on (i) the composition level of home endowments is increasing or not, and (ii) how relative price responds to the changes in C_t. To highlight the composition effects from the changing endowments, CLW (2014) further derive that $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} > 0$ is equivalent to

$$Cov\left(\frac{\phi_i'(C_t)}{\phi_i(C_t)},\ \phi_i(C_t) dy_{it+1} \right) > 0. \tag{6}$$

Inequality (6) shows that for raising aggregate consumption, not only does the change of endowments in certain sector matter, but also its price sensitivity to changes in C_t. Home is more likely to stimulate aggregate consumption if the endowment change in the goods whose price are more manipulable. This result enables us to take a glance at what the optimal policy should look like when the growing endowments are from different sectors. If the growing endowments are oriented in a sector whose price is more sensitive to changes in C_t, Home is more likely to implement optimal allocations with tariffs or capital controls.

Next, following the definition of the wedge in CLW, the pro-cyclical aggregate consumption implies a countercyclical wedges. The wedge, formally defined as $w_t \equiv U'(C_t)/\mu U^{*'}(C_t^*) - 1$, measures the marginal utility difference between domestic and foreign aggregate consumptions.

Corollary 2. *The wedge is lower from period t to period $t+1$, $w_{t+1} < w_t$, if and only if $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} > 0$.*

The implementation of optimal allocation in next subsection will show that the optimal time-varying tariff policy depends on the counter-cyclical behavior of wedge.

2.2 Unilaterally Optimal Good-specific Trade Taxes/Subsidies

This subsection explores how the optimal allocations are implemented with trade taxes/subsidies. In order to facilitate the comparison between different policy instruments (specifically for the capital control policy in the later section), I assume that consumers can trade one-period good-specific bonds on international capital markets. Note that this subsection solely characterizes the optimal trade tariff policy, i.e. no capital control tax on the net asset position in the bond markets.

Correspondingly, I adjust the per-period budget constraint that domestic representative consumer faces. **Let τ_{it} be a time-varying, ad valorem good-specific trade taxes/subsidies imposed by Home country on good i.** Assume foreign country is passive, so that there is no tariff retaliation. The per-period budget constraint of home consumer is now given by

$$p_t \cdot (c_t - y_t) + p^*_{t+1} \cdot B_{t+1} \quad = \quad p^*_t \cdot B_t - T_t \tag{7}$$

where B_t is the vector of current foreign asset positions, T_t is a lump-sum tax, $p_t = p^*_t \circ (1+\tau_t)$ is the price of imported or exported goods of Home, p^*_t is the world prices, $\circ$ is the Hadamard product (entrywise product), which leads to the price for good i is $p^*_{it}(1+\tau_{it})$. In additional, assume a standard no-Ponzi condition holds, $\lim\limits_{t\to\infty} p^*_t \cdot B_t \geq 0$. Note that the tariffs only distort the tradable good prices relative to world prices, while the good-specific bond prices are not affected. In the appendix, I show that equation (7) concludes all possible trade balance/imbalance scenarios.

The first-order conditions associated with utility maximization at home are given by

$$U'(C_t) f_i(c_t) = \beta U'(C_{t+1}) f_i(c_{t+1}) \frac{p^*_{it}}{p^*_{it+1}} \frac{1+\tau_{it}}{1+\tau_{it+1}} \quad \text{for } i = 1,2,\ldots,n \tag{8}$$

where f_i is the first derivative of f with respective to good i. Let $P_t \equiv \min_c \{p_t \cdot c : f(c) \geq 1\}$ denote the home consumer price index[3] at date t. The price index measures the minimum expenditure of

[3] One can formally prove that $P_t = \frac{p_{it}}{f_i(c_t)} = \frac{p^*_{it}(1+\tau_{it})}{f_i(c_t)}$ for all $i = 1,\ldots,n$. Equation (9) is directly derived from this observation and condition (8).

a unit aggregate consumption level at Home. Then we can rewrite above condition into a compact form.

$$U'(C_t) = \beta U'(C_{t+1}) \frac{P_t}{P_{t+1}} \tag{9}$$

Note that the Home's price index is inclusive of the optimal trade taxes, so that any changes in tariffs are shown in the changes of Home price index. Since there is no tariff abroad, same logic implies

$$U^{*'}(C_t^*) = \beta U^{*'}(C_{t+1}^*) \frac{P_t^*}{P_{t+1}^*} \tag{10}$$

where $P_t^* \equiv \min_{c^*} \{p_t^* \cdot c^* : f^*(c^*) \geq 1\}$. Combining conditions (9) and (10) to yield

$$\frac{U'(C_t)}{U^{*'}(C_t^*)} = \frac{U'(C_{t+1})}{U^{*'}(C_{t+1}^*)} \frac{P_t/P_t^*}{P_{t+1}/P_{t+1}^*} \tag{11}$$

Replaced the ratio of the marginal utility of domestic and foreign consumption with the definition of wedge, we can arrange Home's optimal strategy as

$$\frac{P_{t+1}/P_{t+1}^*}{P_t/P_t^*} = \frac{1+w_{t+1}}{1+w_t} \tag{12}$$

Left hand side of equation (12) is the change of Home's real exchange rate, and the right hand side is the wedge change. Therefore, with many goods, Home's optimal tariffs/subsidies are represented as a real exchange rate manipulation, and the direction depends on whether the wedge w_t between the marginal utility of domestic and foreign consumption is increasing or decreasing. According to Corollary 2, the wedge is a decreasing function of home aggregate consumption C_t. Equation

Proof: The associated first-order conditions are given by (i) $p_i = \lambda f_i[c_t(1)]$ and (ii) $f_i[c_t(1)] = 1$. This implies

$$P_t = \sum_i p_{it} c_{it}(1) = \lambda \sum_i f_i[c_{it}(1)] c_{it}(1) = \lambda g[c_t(1)] = \lambda$$

(12) implies Home's optimal trade policy should affects real exchange rate to depreciate due to the counter-cyclical of the optimal wedge.

Proposition 3 (Optimal trade taxes/subsidies)**.** *Suppose that the optimal policy is implemented with trade taxes/subsidies, and between periods t and $t+1$, the domestic endowment has a small change $dy_{t+1} = y_{t+1} - y_t$. Then optimal trade taxes/subsidies make Home's real exchange rate*

1. *depreciate if $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} > 0$;*
2. *appreciate if $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} < 0$;*
3. *unchanged if $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} = 0$.*

Next, I further demonstrate what the optimal tariffs look like across different sectors during the same period. The intra-temporary Euler equation of Home representative agent's problem is

$$\frac{1+\tau_{jt}}{1+\tau_{kt}} = \frac{f_j\left(c(C_t)\right)/p^*_{jt}}{f_k\left(c(C_t)\right)/p^*_{kt}} \quad \text{where } j \neq k \tag{13}$$

Note that equation (13) can also be obtained from the price index at Home, i.e. $P_t = \frac{p_{it}}{f_i(c(C_t))}$, $\forall i$.

From equation (1), we have

$$p^*_{it} = \frac{\beta^t}{\lambda^*} U^{*'}\left(C^*(C_t)\right) f^*_i\left(c^*(C_t)\right) \tag{14}$$

Combining equation (13) and (14) to yield

$$\frac{1+\tau_{jt}}{1+\tau_{kt}} = \frac{f_j\left(c(C_t)\right)/f^*_j\left(c^*(C_t)\right)}{f_k\left(c(C_t)\right)/f^*_k\left(c^*(C_t)\right)} \tag{15}$$

Equation (15) shows that for certain period t, the optimal tax rate on sector i depends on home's relative marginal demand, i.e. $\frac{f_i(c(C_t))}{f^*_i(c^*(C_t))}$.

In order to see how the intratemporal tariffs at sectoral level, we need to specify utility and the demand function form. It is not surprising that the multi-good economy shrinks to one-good economy if both country are symmetric. By decomposing the increasing endowments $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1}$ into inter-temporal and intra-temporal components, Costinot et al. (2014) argues that if both countries have identical utility function and aggregate function, the economy boils down to one-good case. Their claim and analysis is still valid here. More interesting thing is that, under asymmetric demand, if the two-layer utility functions are specified as upper - CRRA and lower - CES, the equation (15) will equates to one, i.e. optimal tax rates on both sectors should be uniform. I elaborate more of this finding in the following subsection.

2.3 An Example with CRRA and CES utility Functions

This subsection focuses on a simple two-layer utility function. I assume the upper-layer utility function follows CRRA, and the lower-layer aggregate consumption function is CES[4].[5] There are two goods, good a and b. Home and Foreign are asymmetric in terms of good perference. Sector a is home-bias, i.e.

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad C = \left[\xi a_1^{\frac{\sigma-1}{\sigma}} + (1-\xi) b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{16}$$

where intertemporal elasticity $\gamma \geq 0$, expenditure share $\xi > 1/2$, and elasticity of substitution $\sigma \geq 0$. Similarly, foreign country's utility functions take the same form, but the roles of good a and b in aggregate consumption are reversed. Namely,

$$U^*(C^*) = \frac{(C^*)^{1-\gamma}}{1-\gamma}, \quad C^* = \left[(1-\xi) a_2^{\frac{\sigma-1}{\sigma}} + \xi b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \tag{17}$$

With above utility functions, an analog proposition like Proposition 5 in CLW(2014) to illustrate the optimal real exchange rate policy can be obtained.

Proposition 4 (Import- vs. export-oriented growth)**.** *Suppose that equations (16) and (17) hold with $\gamma \geq 0$ and $\xi > 1/2$ and that home endowment has a small change $dy_{t+1} = y_{t+1} - y_t$.*

1. *if growth is import oriented, $dy_{at+1} > 0$ and $dy_{bt+1} = 0$, it is optimal to impose good-specific trade taxes/subsidies to depreciate Home's real exchange rate.*
2. *if growth is export oriented, $dy_{at+1} = 0$ and $dy_{bt+1} > 0$, it is optimal to impose good-specific trade taxes/subsidies to depreciate Home's real exchange rate if and only if*

$$\gamma > \frac{-\xi(1-\delta) a_{1t}^{\frac{\sigma-1}{\sigma}} (Y_a - a_{1t})}{\sigma\left[\delta Y_a C_t^{\frac{\sigma-1}{\sigma}} + (1-\delta)\xi a_{1t}^{\frac{\sigma-1}{\sigma}+1}\right]}, \quad \delta = \left(\frac{\xi}{1-\xi}\right)^{2\sigma}$$

[4] CLW (2014) utilize Cobb-Douglas as the form of aggregate consumption. Note that Cobb-Douglas function is just a special case of CES with elasticity of substitution $\sigma = 1$.

[5] To simplify the notation, in this subsection, I am going to use Good a and b to denote the two goods, and subscript 1 denotes for Home and 2 for Foreign country, i.e. $c_t = (a_{1t}, b_{1t})'$, and $c_t^* = (a_{2t}, b_{2t})'$.

Proposition 4 illustrates the results of inter- and intra-temporary effects from the growing endowment. If the increasing endowment comes from the import-oriented sector, then the aggregate consumption must be pushed up. Since next period the endowment of good a increases, the price of good a at period $t+1$ is lower than at period t. Home is expected to have a less trade deficit or a larger trade surplus on sector a. Either way gives Home more incentive to push up the price by subsidizing the aggregate consumption. The intratemporay effects implies relative price of good a to good b will decrease too, which induces next period aggregate consumption increases, and then promotes intratemporay price of good a. Intertemporay and intratemporay effects works in the same directions.

If the growing endowment is a boost in export-oriented sector, then the direction of change in aggregate consumption is ambiguous. The inter- and intra-temporal effects from the increasing endowment have opposite effects on C_t. The intertemporay effect promotes the aggregate consumption in the same way as the previous case, while not for the intratemporay effect. The relative price tomorrow of good b to good a increases, which reduces the consumption on good b. Thus the aggregate consumption decreases. Proposition 4 further shows that under the growth of export-oriented sector, I find the intertemporay effects would dominate the intratemporay effects if and only if the intertemporal elasticity of substitution is big enough.

More interestingly, with the CRRA and CES utility function form, the unilaterally optimal tariffs on both sectors are identical.[6]

Proposition 5 (Optimal tariffs across different sectors)**.** *Suppose that the utility functions are specified by (16) and (17) with $\gamma \geq 0$ and $\xi > 1/2$, optimal policy is implemented with trade taxes/subsidies. Then optimal trade tax rate is identical on both sectors.*

Proof: see appendix.

[6]This result is still valid for the example with CRRA and Cobb-Douglas utility form. See the proof in the appendix.

3 Trade Policies and Capital Controls

To see the interaction between trade policy and capital control policy in the economy, in addition to trade taxes/subsidies, I include a time-varying tax, κ_t, on gross return on net lending in all bond markets, as in CLW. Then, the representative consumer at Home has the maximization problem.

$$\max_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} \quad \sum_{t=0}^{\infty} \beta^t U(C_t) \tag{18}$$

$$\text{s.to} \quad p_t \cdot (c_t - y_t) + p_{t+1}^* \cdot B_{t+1} = (1 - \kappa_{t-1})(p_t^* \cdot B_t) - T_t \tag{19}$$

where $C_t = f(c_t)$, $p_t = p_t^* \circ (1 + \tau_t)$, and the lump-sum transfer is set as $T_t = -\tau_t \cdot (c_{it} - y_{it}) - \kappa_{t-1}(p_t^* \cdot B_t)$. Again, assume a standard no-Ponzi condition holds, $\lim_{t \to \infty} p_t^* \cdot B_t \geq 0$. The first-order condition associated with utility maximization at home is given by

$$\frac{U'(C_t)}{\beta U'(C_{t+1})} = \frac{f_i(c_{t+1})}{f_i(c_t)} \frac{p_{it}^*(1+\tau_{it})}{p_{it+1}^*(1+\tau_{it+1})}(1-\kappa_t) \quad \text{for } i = 1, 2, \ldots, n \tag{20}$$

$$\text{or} \quad \frac{U'(C_t)}{\beta U'(C_{t+1})} = \frac{f_i(c_{t+1})}{f_i(c_t)} \frac{p_{it}^*}{p_{it+1}^*} Z_{it} \quad \text{for } i = 1, 2, \ldots, n \tag{21}$$

where $Z_{it} = \frac{1+\tau_{it}}{1+\tau_{it+1}}(1-\kappa_t)$ is a mixed-policy variable aggregating the effects of trade taxes and capital flow taxes to good i at period t. By taking the tax structure as a whole, Z, its expression simply implies some relationship between trade taxes and capital controls along the optimal path.

Implication 1: same Z-value can result in same equilibriums even when the components of policy mix are different. Suppose Home's mixed-policy value $Z_{it} = \frac{1+\tau_{it}}{1+\tau_{it+1}}(1-\kappa_t) = z_{i1}$ for some $z_{i1} \in \mathbb{R}$, and there is an alternative mixed-policy with $Z_{it}^{'} = \frac{1+\tau_{it}^{'}}{1+\tau_{it+1}^{'}}(1-\kappa_t^{'}) = z_{i2}$ for some $z_{i2} \in \mathbb{R}$ with $\tau_{it}^{'} \neq \tau_{it}$, then their equilibrium allocations would be same if $z_{i1} = z_{i2}$ for all i and their initial conditions are same. Two extreme cases are interesting. First, to replicate the equilibrium with optimal capital controls in CLW (2014), instead of taxing capital inflows/subsidizing capital outflows when $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} > 0$, one can solely implement tariffs with same decreasing rate to all sectors, i.e. $\frac{1+\tau_{it+1}}{1+\tau_{it}} = \frac{1}{1-\kappa_t} < 1$. Second, when the Z-value equals to one, trade taxes offset the effects of capital controls, thus no price distortion.

Implication 2: the subscripts in mixed-policy variable represent the properties of those two policy instruments: capital controls is a more blunt instrument that affects all sectors, while trade taxes can be tailored to any specific sectors. A negative κ_t, meaning taxes capital inflow/subsidizing capital outflow, has same effects with reducing the gross tariffs/increasing subsidies on all sectors at a same rate. Since any Z-value consisted of trade taxes and capital controls can be equaled by a Z-value with another tariff-only sequence, $\{\tau_{it}\}$, we might expect that if the optimal policy can implemented with trade taxes/subsidies, then it is optimal to not distort capital flows.

The following proposition formally summarizes the above implications.

Proposition 6 (Interactions between trade policy and capital controls)**.** *Any real allocation that can be achieved with capital controls can also be implemented with good-specific tariffs. A negative κ_t, meaning taxing capital inflow/subsidizing capital outflow, is equivalent of a uniform reduction in gross trade tariffs.*

Proposition 6 shows an equivalence result: a uniform changing in trade policy can achieve the same outcome of capital control policy. This equivalence resembles the finding of Jeanne (2012): capital controls can achieve the same outcomes as trade protectionism. However, since there is only one tradable good in his model, his conclusion might be inaccurate. Introducing multiple goods would lead to the other inversed result.

Corollary 7. *If Government can get access to good-specific trade taxes, then capital controls is not necessary ($\kappa_t = 0$).*

The Corollary 7 coincides with the finding of Benigno et al. (2016): when government can easily get access to the real exchange rate policy, there is no need for restrict the international capital flow. To illustrate more on the channel of how optimal trade taxes and optimal capital flow taxes interact, we can further rewrite the intertemporal Euler condition (20) in a compact way, same as before.

$$\frac{U'(C_t)}{\beta U'(C_{t+1})} = \frac{P_t}{P_{t+1}}(1-\kappa_t) \tag{22}$$

Although condition (22) is identical as the one in CLW, the Home's price index here is inclusive of trade taxes. Since there is no tariff or capital tax retaliation abroad, we can have foreign country's first-order condition

$$U^{*'}(C_t^*) = \beta U^{*'}(C_{t+1}^*)\frac{P_t^*}{P_{t+1}^*} \tag{23}$$

Combining condition (22) and (23) and deriving the optimal capital controls to yield

$$\kappa_t = 1-\left(\frac{1+w_t}{1+w_{t+1}}\right)\left(\frac{P_{t+1}/P_{t+1}^*}{P_t/P_t^*}\right) \tag{24}$$

This expression for optimal capital controls is identical with the one in CLW, except that the Home's price indices here include the effects of tariffs. With many goods, the value of κ_t depends on (i) whether the wedge w_t is increasing or decreasing and (ii) whether Home's real exchange rate inclusive of trade taxes appreciates or depreciates between t and $t+1$. According to the Proposition 3, we know optimal tariffs can manipulate the real exchange rate by correctly creating the wedge changes. Thus, equation (24) implies a zero optimal capital flow taxes.

4 Concluding Remarks

Under the dynamic two-country multi-good endowment economy in CLW, this paper shows that home has incentive to use unilaterally optimal trade taxes/subsidies to support its real exchange rate in its favor. Specially, when the demand is specified as CRRA and CES, the optimal tariff rate is uniform across sectors in a two-good economy. The optimal capital control policy in CLW is equivalent to a uniform change in trade taxes/subsides on all sectors. Furthermore, if the government can get access to good-specific trade taxes/subsides, it is unnecessary to control international capital flows.

The results shown are still incomplete and more work need to be done. For example, some quantitative exercise of the concrete example can be done to illustrate the mechanism of these intertemporal and intratemporal effects.

Appendix

Derivation of Equation (7)

1. (Trade Deficit) Suppose Home country runs a trade deficit on good i at period t, i.e. $c_{it} > y_{it}$. Good i produced domestically is sold at a price of p^*_{it} (same as world price), while imported good i is charged at a price of $p^*_{it}(1+\tau_{it})$.

$$p^*_{it}c^d_{it} + p_{it}c^f_{it} + p^*_{it+1}B_{it+1} \quad = \quad p^*_{it}y_{it} + p^*_{it}B_{it} - T_t$$

   where c^d_{it} is the consumption on good i of domestic part, c^f_{it} is the excess demand, or the consumption on good i of imported foreign part, and $c^d_{it} + c^f_{it} = c_{it}$. Due to the low price of domestic product, domestic good will be sold out prior to the imported good, i.e. $c^d_{it} = y_{it}$ and $c^f_{it} = c_{it} - y_{it}$. Hence, the budget constraint can be rewritten as

$$p_{it}(c_{it} - y_{it}) + p^*_{it+1}B_{it+1} \quad = \quad p^*_{it}B_{it} - T_t \tag{25}$$

2. (Trade Surplus) Suppose Home country runs a trade surplus on good i at period t, i.e. $c_{it} < y_{it}$. Good i produced domestically is sold at a price of p^*_{it}, while exported part of good i receives a subsidy, $p_{it} = p^*_{it}(1+\tau_{it})$.

$$p^*_{it}c_{it} + p^*_{it+1}B_{it+1} \quad = \quad p^*_{it}y^d_{it} + p_{it}y^f_{it} + p^*_{it}B_{it} - T_t$$

   where $y^d_{it} = c_{it}$ is the endowment of good i for domestic consumption, and $y^f_{it} = y_{it} - c_{it}$ is the export quantity. Rewrite the equation to yield

$$p_{it}(c_{it} - y_{it}) + p^*_{it+1}B_{it+1} \quad = \quad p^*_{it}B_{it} - T_t \tag{26}$$

3. (Trade Balance) Suppose Home country runs a trade surplus on good i at period t, i.e. $c_{it} = y_{it}$,

then equation (7) obviously includes this case.

In summary, the per-period budget constraint (7) concludes all possible scenarios Home may face.

Proof of Proposition 3

Since the wedge $w_t \equiv \frac{U'(C_t)}{\mu U^{*'}(C_t^*(C_t))} - 1$, U and U^* are concave and C^* is decreasing in C_t along the Pareto frontier. From Proposition 1, we can obtain

$$w_{t+1} < w_t \quad \text{if and only if} \quad \sum_i \frac{\partial \phi_i(C_t)}{\partial C_t} dy_{it+1} > 0.$$

Proposition 3 derive from the above observation and equation (12).

Proof of Proposition 4

Suppose that $dy_{i1+t} > 0$ and $dy_{jt+1} = 0$. Then, $\frac{\partial \phi(C_t)}{\partial C_t} \cdot dy_{t+1} > 0$, where $\phi_t(C_t) \equiv U^*(C^*(C_t))\nabla f^*(c^*(C_t))$ is equivalent to

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} + \left[\frac{f_{ii}^*(c_t^*)}{f_i^*(c_t^*)} \frac{\partial c_i^*(C_t)}{\partial C_t} + \frac{f_{ij}^*(c_t^*)}{f_i^*(c_t^*)} \frac{\partial c_j^*(C_t)}{\partial C_t} \right] > 0 \tag{27}$$

We first simplify the first term in (27). The envelop condition from the definition (2) implies that

$$f_i^*(c_t^*) = -C^{*'}(C_t) f_i(c_t) \tag{28}$$

Since the foreign utility function is CRRA, the first term in (27) can be rewriten as

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))} \frac{\partial C^*(C_t)}{\partial C_t} = \frac{\gamma f_{it}^*}{C_t^* f_{it}} \tag{29}$$

Let us focus on the second term in inequality (27). Since $C^* = \left[(1-\xi)a_1^{\frac{\sigma-1}{\sigma}} + \xi b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$, we can get

$$\frac{f_{11}^*(c_t^*)}{f_1^*(c_t^*)} = \frac{(1-\xi)a_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]} - \frac{1}{\sigma a_{2t}}, \quad \frac{f_{12}^*(c_t^*)}{f_1^*(c_t^*)} = \frac{\xi b_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]},$$

$$\frac{f_{21}^*(c_t^*)}{f_2^*(c_t^*)} = \frac{(1-\xi)a_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}, \quad \frac{f_{22}^*(c_t^*)}{f_2^*(c_t^*)} = \frac{\xi b_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]} - \frac{1}{\sigma b_{2t}}$$

Then, the second term can be rewritten as following

$$\begin{aligned}
\frac{f_{11}^*(c_t^*)}{f_1^*(c_t^*)}\frac{\partial a_2(C_t)}{\partial C_t} + \frac{f_{12}^*(c_t^*)}{f_1^*(c_t^*)}\frac{\partial b_2(C_t)}{\partial C_t} &= \left(\frac{(1-\xi)a_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]} - \frac{1}{\sigma a_{2t}}\right)\frac{\partial a_2(C_t)}{\partial C_t} + \frac{\xi b_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial b_2(C_t)}{\partial C_t} \\
&= -\frac{\xi b_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_2^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial \ln a_2(C_t)}{\partial C_t} + \frac{\xi b_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial \ln b_2(C_t)}{\partial C_t} \\
&= \frac{\xi b_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial \ln[b_2(C_t)/a_2(C_t)]}{\partial C_t} \qquad (30)
\end{aligned}$$

$$\begin{aligned}
\frac{f_{21}^*(c_t^*)}{f_2^*(c_t^*)}\frac{\partial a_2(C_t)}{\partial C_t} + \frac{f_{22}^*(c_t^*)}{f_2^*(c_t^*)}\frac{\partial b_2(C_t)}{\partial C_t} &= \frac{(1-\xi)a_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial a_2(C_t)}{\partial C_t} + \left(\frac{\xi b_{2t}^{-\frac{1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]} - \frac{1}{\sigma b_{2t}}\right)\frac{\partial b_2(C_t)}{\partial C_t} \\
&= \frac{(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial \ln a_2(C_t)}{\partial C_t} - \frac{(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial b_2(C_t)}{\partial C_t} \\
&= -\frac{(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}} + \xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{\partial \ln[b_2(C_t)/a_2(C_t)]}{\partial C_t} \qquad (31)
\end{aligned}$$

Next we compute $\frac{\ln[b_2(C_t)/a_2(C_t)]}{\partial C_t}$, By the definition, $c(C_t)$ and $c^*(C_t)$ are the solution of

$$\max_{c,c^*} \quad \left[(1-\xi)a_2^{\frac{\sigma-1}{\sigma}} + \xi b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}$$

subject to

$$a_1 + a_2 \leq Y_a$$

$$b_1 + b_2 \leq Y_b$$

$$\left[\xi a_1^{\frac{\sigma-1}{\sigma}} + (1-\xi)b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \geq C_t$$

Plugging the first two resource constraints into the objective, and let λ be the lagrangian multiplier of the third constraint, we have the lagrangian

$$\mathscr{L} = \left[(1-\xi)(Y_a - a_1)^{\frac{\sigma-1}{\sigma}} + \xi(Y_b - b_2)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} + \lambda\left(-C_t + \left[\xi a_1^{\frac{\sigma-1}{\sigma}} + (1-\xi)b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}\right)$$

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$$\begin{aligned}
\frac{\xi(Y_b - b_1)^{-\frac{1}{\sigma}}}{(1-\xi)(Y_a - a_1)^{-\frac{1}{\sigma}}} &= \frac{(1-\xi)b_1^{-\frac{1}{\sigma}}}{\xi a_1^{-\frac{1}{\sigma}}} \\
\frac{b_2(C_t)}{a_2(C_t)} &= \delta\left[\frac{Y_b - b_2(C_t)}{Y_a - a_2(C_t)}\right], \quad \delta = \left(\frac{\xi}{1-\xi}\right)^{2\sigma} \qquad (32)
\end{aligned}$$

According to the Home's aggregate consumption function, we can obtain

$$\begin{aligned}
\left[\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}} + (1-\xi)b_1(C_t)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} &= C_t \\
\xi + (1-\xi)\left[\frac{Y_b - b_2(C_t)}{Y_a - a_2(C_t)}\right]^{\frac{\sigma-1}{\sigma}} &= \left[\frac{C_t}{Y_a - a_2(C_t)}\right]^{\frac{\sigma-1}{\sigma}} \\
\frac{Y_b - b_2(C_t)}{Y_a - a_2(C_t)} &= \left[\left(\left[\frac{C_t}{Y_a - a_2(C_t)}\right]^{\frac{\sigma-1}{\sigma}} - \xi\right)\Big/(1-\xi)\right]^{\frac{\sigma}{\sigma-1}} \qquad (33)
\end{aligned}$$

Combining equation (32) and equation (33) to yield

$$\begin{aligned}
\frac{b_2(C_t)}{a_2(C_t)} &= \delta \left[\left(\left[\frac{C_t}{Y_a - a_2(C_t)} \right]^{\frac{\sigma-1}{\sigma}} - \xi \right) / (1-\xi) \right]^{\frac{\sigma}{\sigma-1}} \\
\ln \left[\frac{b_2(C_t)}{a_2(C_t)} \right] &= \ln \delta + \frac{\sigma}{\sigma - 1} \left[\ln \left(\left[\frac{C_t}{a_1(C_t)} \right]^{\frac{\sigma-1}{\sigma}} - \xi \right) - \ln(1-\xi) \right] \\
\frac{\partial \ln [b_2(C_t)/a_2(C_t)]}{\partial C_t} &= \frac{\sigma}{\sigma-1} \frac{1}{\left[\frac{C_t}{a_1(C_t)}\right]^{\frac{\sigma-1}{\sigma}} - \xi} \frac{\sigma - 1}{\sigma} \left[\frac{C_t}{a_1(C_t)} \right]^{-\frac{1}{\sigma}} \left[\frac{1}{a_1(C_t)} - \frac{C_t}{a_1^2(C_t)} \frac{\partial a_1(C_t)}{\partial C_t} \right] \\
\frac{\partial \ln [b_2(C_t)/a_2(C_t)]}{\partial C_t} &= \frac{\left[\frac{C_t}{a_1(C_t)}\right]^{-\frac{1}{\sigma}}}{\left[\frac{C_t}{a_1(C_t)}\right]^{\frac{\sigma-1}{\sigma}} - \xi} \frac{1}{a_1(C_t)} \left[1 - C_t \frac{\partial \ln a_1(C_t)}{\partial C_t} \right] \qquad (34)
\end{aligned}$$

Let us compute $\frac{\partial \ln a_1(C_t)}{\partial C_t}$. From equation (32) we have

$$\begin{aligned}
\frac{Y_b - b_1(C_t)}{Y_a - a_1(C_t)} &= \delta \left[\frac{b_1(C_t)}{a_1(C_t)} \right] \\
b_1(C_t) &= \frac{Y_b a_1(C_t)}{\delta Y_a + (1-\delta) a_1(C_t)} \qquad (35)
\end{aligned}$$

Due to the Home's aggregate consumption function, we can obtain

$$b_1(C_t) = \left[\frac{1}{1-\xi} \left(C_t^{\frac{\sigma-1}{\sigma}} - \xi a_1^{\frac{\sigma-1}{\sigma}} \right) \right]^{\frac{\sigma}{\sigma-1}} \qquad (36)$$

Equating equation (35) and (36), taking the log and differentiating, we can get

$$
\begin{aligned}
\frac{Y_b a_1(C_t)}{\delta Y_a + (1-\delta) a_1(C_t)} &= \left[\frac{1}{1-\xi}\left(C_t^{\frac{\sigma-1}{\sigma}} - \xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}\right)\right]^{\frac{\sigma}{\sigma-1}} \\
\ln Y_b + \ln a_1(C_t) - \ln[\delta Y_a + (1-\delta) a_1(C_t)] &= \frac{\sigma}{\sigma-1}\left[-\ln(1-\xi) + \ln\left(C_t^{\frac{\sigma-1}{\sigma}} - \xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}\right)\right] \\
\frac{\partial \ln a_1(C_t)}{\partial C_t} - \frac{(1-\delta) a_1(C_t)}{[\delta Y_a + (1-\delta) a_1(C_t)]}\frac{\partial \ln a_1(C_t)}{\partial C_t} &= \frac{\sigma}{\sigma-1}\frac{\left[\frac{\sigma-1}{\sigma} C_t^{-\frac{1}{\sigma}} - \frac{\sigma-1}{\sigma}\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}\frac{\partial \ln a_1(C_t)}{\partial C_t}\right]}{C_t^{\frac{\sigma-1}{\sigma}} - \xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}} \\
\left[1 - \frac{(1-\delta) a_1(C_t)}{\delta Y_a + (1-\delta) a_1(C_t)} + \frac{\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}}{C_t^{\frac{\sigma-1}{\sigma}} - \xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}}\right]\frac{\partial \ln a_1(C_t)}{\partial C_t} &= \frac{C_t^{-\frac{1}{\sigma}}}{C_t^{\frac{\sigma-1}{\sigma}} - \xi a_1(C_t)^{\frac{\sigma-1}{\sigma}}} \\
\left[\delta Y_a C_t^{\frac{\sigma-1}{\sigma}} + (1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}\right]\frac{\partial \ln a_1(C_t)}{\partial C_t} &= C_t^{-\frac{1}{\sigma}}[\delta Y_a + (1-\delta) a_1(C_t)] \\
\frac{\partial \ln a_1(C_t)}{\partial C_t} &= \frac{C_t^{-\frac{1}{\sigma}}[\delta Y_a + (1-\delta) a_1(C_t)]}{\delta Y_a C_t^{\frac{\sigma-1}{\sigma}} + (1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}} \qquad (37)
\end{aligned}
$$

Plugging equation (37) into (34) to yield

$$
\begin{aligned}
\frac{\partial \ln[b_2(C_t)/a_2(C_t)]}{\partial C_t} &= \frac{\left[\frac{C_t}{a_1(C_t)}\right]^{-\frac{1}{\sigma}}}{\left[\frac{C_t}{a_1(C_t)}\right]^{\frac{\sigma-1}{\sigma}} - \xi}\frac{1}{a_1(C_t)}\left[1 - \frac{C_t^{\frac{\sigma-1}{\sigma}}[\delta Y_a + (1-\delta) a_1(C_t)]}{\delta Y_a C_t^{\frac{\sigma-1}{\sigma}} + (1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}}\right] \\
&= \frac{\left[\frac{C_t}{a_1(C_t)}\right]^{-\frac{1}{\sigma}}}{(1-\xi)\left[\frac{b_1(C_t)}{a_1(C_t)}\right]^{\frac{\sigma-1}{\sigma}}}\ \frac{(1-\delta)\left[\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}} - C_t^{\frac{\sigma-1}{\sigma}}\right]}{\delta Y_a C_t^{\frac{\sigma-1}{\sigma}} + (1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}} \\
&= -\frac{(1-\delta) C_t^{-\frac{1}{\sigma}} a_1(C_t)}{\delta Y_a C_t^{\frac{\sigma-1}{\sigma}} + (1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}} \qquad (38)
\end{aligned}
$$

Note that $\delta = \left(\frac{\xi}{1-\xi}\right)^{2\sigma}$. Since $\xi > 1/2$, then $\delta > 1$. Consider the sign of the denominator, since $\delta > |1-\delta|$, $Y_a > a_1$, and $C_t^{\frac{\sigma-1}{\sigma}} > \xi a_1^{\frac{\sigma-1}{\sigma}}$, we know the denominator is positive. Thus, $\frac{\ln[b_2(C_t)/a_2(C_t)]}{\partial C_t} > 0$.

First note that equation (38) and (30) imply

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))}\frac{\partial C^*(C_t)}{\partial C_t}+\left[\frac{f^*_{11}(c^*_t)}{f^*_1(c^*_t)}\frac{\partial a_2(C_t)}{\partial C_t}+\frac{f^*_{12}(c^*_t)}{f^*_1(c^*_t)}\frac{\partial b_2(C_t)}{\partial C_t}\right]$$
$$=\frac{\gamma f^*_{1t}}{C^*_t f_{1t}}+\frac{\xi b_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}}+\xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{-(1-\delta)C_t^{-\frac{1}{\sigma}}a_1(C_t)}{\delta Y_a C_t^{\frac{\sigma-1}{\sigma}}+(1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}}>0 \tag{39}$$

Thus, if $dy_{1t+1}>0$ and $dy_{2t+1}=0$, then $\kappa_t<0$.

Second, note that equation (38) and (31) imply

$$\frac{U^{*\prime\prime}(C^*(C_t))}{U^{*\prime}(C^*(C_t))}\frac{\partial C^*(C_t)}{\partial C_t}+\left[\frac{f^*_{21}(c^*_t)}{f^*_2(c^*_t)}\frac{\partial a_2(C_t)}{\partial C_t}+\frac{f^*_{22}(c^*_t)}{f^*_2(c^*_t)}\frac{\partial b_2(C_t)}{\partial C_t}\right]$$
$$=\frac{\gamma f^*_{1t}}{C^*_t f_{1t}}+\frac{(1-\xi) a_{2t}^{\frac{\sigma-1}{\sigma}}}{\sigma\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}}+\xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]}\frac{(1-\delta)C_t^{-\frac{1}{\sigma}}a_1(C_t)}{\delta Y_a C_t^{\frac{\sigma-1}{\sigma}}+(1-\delta)\xi a_1(C_t)^{\frac{\sigma-1}{\sigma}+1}} \tag{40}$$

By the definition of aggregate consumption, we have

$$f_{1t}=f_1^{'}(c_t)=\frac{\xi C_t}{a_{1t}^{1/\sigma}\left[\xi a_{1t}^{\frac{\sigma-1}{\sigma}}+(1-\xi)b_{1t}^{\frac{\sigma-1}{\sigma}}\right]} \tag{41}$$

$$f^*_{1t}=f_1^{*'}(c^*_t)=\frac{(1-\xi)C^*_t}{a_{2t}^{1/\sigma}\left[(1-\xi)a_{2t}^{\frac{\sigma-1}{\sigma}}+\xi b_{2t}^{\frac{\sigma-1}{\sigma}}\right]} \tag{42}$$

Let equation (40) greater than 0, and simplifying with equation (41) and (42), we can get

$$\gamma>\frac{-\xi(1-\delta)a_{1t}^{\frac{\sigma-1}{\sigma}}(Y_a-a_{1t})}{\sigma\left[\delta Y_a C_t^{\frac{\sigma-1}{\sigma}}+(1-\delta)\xi a_{1t}^{\frac{\sigma-1}{\sigma}+1}\right]} \tag{43}$$

Proof of Proposition 5: Uniform tariff

Since the aggregate consumptions are defined as

$$\begin{aligned} f(c_t) &= \left[\xi a_1^{\frac{\sigma-1}{\sigma}} + (1-\xi) b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \\ f^*(c_t^*) &= \left[(1-\xi) a_2^{\frac{\sigma-1}{\sigma}} + \xi b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

we can have the first derivatives

$$\begin{aligned} f_{1t} = f_1^{'}(c_t) &= \xi \left[\xi a_1^{\frac{\sigma-1}{\sigma}} + (1-\xi) b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} a_1^{-\frac{1}{\sigma}} \\ f_{2t} = f_2^{'}(c_t) &= (1-\xi) \left[\xi a_1^{\frac{\sigma-1}{\sigma}} + (1-\xi) b_1^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} b_1^{-\frac{1}{\sigma}} \\ f_{1t}^* = f_1^{*'}(c_t^*) &= (1-\xi) \left[(1-\xi) a_2^{\frac{\sigma-1}{\sigma}} + \xi b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} a_2^{-\frac{1}{\sigma}} \\ f_{2t}^* = f_2^{*'}(c_t^*) &= \xi \left[(1-\xi) a_2^{\frac{\sigma-1}{\sigma}} + \xi b_2^{\frac{\sigma-1}{\sigma}}\right]^{\frac{1}{\sigma-1}} b_2^{-\frac{1}{\sigma}} \end{aligned}$$

From equation (15), we have

$$\begin{aligned} \frac{1+\tau_{at}}{1+\tau_{bt}} &= \frac{f_{1t}/f_{1t}^*}{f_{2t}/f_{2t}^*} \\ &= \frac{\xi}{1-\xi}\left(\frac{a_1}{b_1}\right)^{-\frac{1}{\sigma}} \frac{\xi}{1-\xi}\left(\frac{b_2}{a_2}\right)^{-\frac{1}{\sigma}} \\ &= \left(\frac{\xi}{1-\xi}\right)^2 \left(\frac{a_1}{b_1}\cdot\frac{b_2}{a_2}\right)^{-\frac{1}{\sigma}} \\ &= \left(\frac{\xi}{1-\xi}\right)^2 \left(\frac{a_1}{b_1}\cdot\delta\left[\frac{Y_b-b_2}{Y_a-a_2}\right]\right)^{-\frac{1}{\sigma}} \\ &= 1 \end{aligned}$$

where the last two rows are due to the optimal conditions (32). Hence, the optimal tariffs ob both sectors should be identical, i.e. $\tau_{at} = \tau_{bt}$.

CRRA and Cobb-Douglas example in CLW (2014): Uniform tariff proof

This is the simple two-good asymmetric example in CLW, I am going to illustrate how relative tariff changes due to a domestic endowment growing. Assume that both utility functions follow constant relative risk aversion (CRRA), aggregate consumption functions are Cobb-Douglas, there are two goods, a and b, and Sector a is home-bias, i.e.

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma}, \quad C = a_1^{\alpha} b_1^{1-\alpha} \tag{44}$$

$$U^*(C^*) = \frac{C^{*1-\gamma}}{1-\gamma}, \quad C^* = a_2^{1-\alpha} b_2^{\alpha} \tag{45}$$

where the aggregate functions are asymmetric, $\alpha > 1/2$, $\gamma \geq 0$ is the risk aversion, and consumption vectors $c = [a_1, b_1]'$, $c^* = [a_2, b_2]'$.

First, we can have the first derivatives

$$\begin{aligned}
f_{1t} = f_1'(c_t) &= \alpha a_{1t}^{\alpha-1} b_{1t}^{1-\alpha} \\
f_{2t} = f_2'(c_t) &= (1-\alpha) a_{1t}^{\alpha} b_{1t}^{-\alpha} \\
f_{1t}^* = f_1^{*'}(c_t^*) &= (1-\alpha) a_{2t}^{-\alpha} b_{2t}^{\alpha} \\
f_{2t}^* = f_2^{*'}(c_t^*) &= \alpha a_{2t}^{\alpha} b_{2t}^{\alpha-1}
\end{aligned}$$

From equation (15), we have

$$\begin{aligned}
\frac{1+\tau_{at}}{1+\tau_{bt}} &= \frac{f_{1t}/f_{1t}^*}{f_{2t}/f_{2t}^*} \\
&= \left(\frac{\alpha}{1-\alpha}\right)^2 \left(\frac{a_2}{b_2} \cdot \frac{b_1}{a_1}\right)
\end{aligned} \tag{46}$$

Similarly, from the problem, i.e.

$$
\begin{aligned}
\max_{c,c^*} \quad & a_2^{1-\alpha} b_2^{\alpha} \\
\text{subject to} \quad & a_1 + a_2 \leq Y_a \\
& b_1 + b_2 \leq Y_b \\
& a_1^{\alpha} b_1^{1-\alpha} \geq C_t
\end{aligned}
$$

we can obtain a optimal condition, i.e.

$$\frac{b_2}{a_2} = \left(\frac{\alpha}{1-\alpha}\right)^2 \frac{b_1}{a_1}$$

Combined with equation (46), the above equation indicates that $\frac{1+\tau_{at}}{1+\tau_{bt}} = 1$, i.e. the sectoral tariff is uniform on sector a and b.

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